

CHARGE CONJUGATION, SIGNATURE AND TWISTS *

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The distinction between charge conjugation and signature in the analysis of quark diagrams is discussed. The signature twist is different from the quark-line twist (which, for mesons, is related to charge conjugation). They coincide only in the planar level.

“Is the twist on a meson line related to charge conjugation or to signature?” This question is a well-known source of confusion. We hope to clarify it in this paper.

Let us consider a one-boundary amplitude with n external mesons which have a definite order along the boundary (fig. 1). We do not assume anything concerning the number of handles. Therefore, this amplitude may represent the planar amplitude [1] with the specified order (namely, the $b = 1, h = 0$ part of the n -point function). The amplitude may also represent any arbitrary quark diagram, or a sum over many quark diagrams, with $b = 1$. (The quark diagrams can be defined as in ref. [2]. We start with particle diagrams, whose lines represent hadrons, and then draw the corresponding quark diagrams. The particle diagrams may be multi-Regge diagrams, the diagrams of the dual models, ϕ^3 diagrams where the field ϕ represents a meson, reggeon-field-theory diagrams, etc.)

We assume that the spectrum of the external mesons is given by $b = 1$ diagrams. Assuming $SU(N)$ invariance, all mesons appear in N^2 degenerate multiplets (only when we include the $b = 2$ terms of the propagator, the $SU(N)$ singlet is not degenerate with the $(N^2 - 1)$ multiplet). We specify the states of the external mesons in the following way: The letter D , for example, in fig. 1 stands for the type of multiplet (e.g., the pseudo-scalar nonet for $N = 3$). It does not specify the member of the multiplet. This is done by using the quark indices. $D_{i, \bar{m}}$ represents the $q_i \bar{q}_{\bar{m}}$ member of the D multiplet, where i and m run over the N flavors. Once we assume the N^2 degeneracy of the meson multiplets, this representation is possible irrespectively of the existence of real quarks. A complete specification of the external meson state is given by $|D_{i, \bar{m}} p, h\rangle$ where p and h stand for the momentum and helicity of

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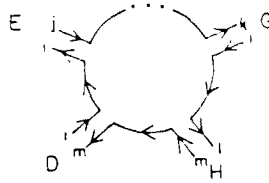


Fig. 1. An ordered $b = 1$ amplitude ($A_{DE \dots GH}$).

the meson. We now have to specify the relation between the (i, \bar{m}) and the (m, \bar{i}) members of the same multiplet. We adopt the following convention:

$$C|D_{i, \bar{m}}(p, h)\rangle = C_D |D_{m, \bar{i}}(p, h)\rangle. \quad (1)$$

where C is the charge-conjugation operator. By taking $i = m$ we see that C_D is the charge-conjugation eigenvalue of the N neutral states $D_{i, \bar{i}}$. (Note that C_D is independent of i and m .) In appendix A we demonstrate eq. (1) assuming that mesons are actually made of quarks. It is clear that any convention which is consistent with a naive theory of real quarks, can be used for a general theory (with or without quarks) which satisfies $SU(N)$ and charge-conjugation invariance on the hadronic level. In our convention (1) the relative phase between the i, \bar{m} and the m, \bar{i} members of the multiplet is determined by charge conjugation. This differs from the usual convention, in which the phases are determined by requiring standard matrix elements of the $SU(N)$ generators sandwiched between two states (see, e.g., ref. [3]). Thus, in ref. [3], $C|\pi^0\rangle = +|\pi^0\rangle$ but $C|\pi^+\rangle = -|\pi^-\rangle$, whereas in our notation $C|\pi^+\rangle = +|\pi^-\rangle$ where π^+ and π^- denote the $u\bar{d}$ and the $d\bar{u}$ members of the multiplet. The usual convention is useful when one has to use Clebsch-Gordan coefficients. The advantage of the quark-diagram approach is that these coefficients are unnecessary. The complete information concerning all $SU(N)$ relations is contained in the following statement: The amplitude of fig. 1 is independent of the indices i, j, \dots, k, l, m . Namely, if we replace the index l , for example, by l' , the amplitude (which now corresponds to a different process) is unchanged. As long as we work with the quark basis (namely, the (i, \bar{m}) states), the amplitude for any given process is just the sum over all quark diagrams which are consistent with the flavor assignment of the external mesons. The coefficient of each term is one. Only when we use the "physical" basis (e.g., $\pi^0 = \sqrt{\frac{1}{2}}[u\bar{u} - d\bar{d}]$) we have to take the appropriate linear combinations, which is equivalent to using the Chan-Paton factors [4]. Another advantage of our convention is that the $SU(N)$ singlet of the D multiplet is given by $(1/\sqrt{N}) \sum_{i=1}^N |D_{i, \bar{i}}\rangle$ (in the convention of ref. [3] $\eta = \sqrt{\frac{1}{2}}(u\bar{u} - d\bar{d})$).

In general, there are $n!$ ways of connecting the quark lines of the n external mesons. Since we are considering the $b = 1$ amplitude, there are only $(n - 1)!$ ways. Fig. 1 represents one of them. If N is large enough, we can choose all the indices

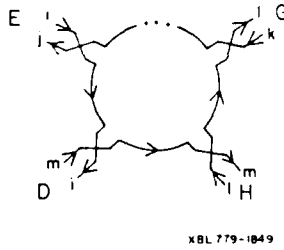


Fig. 2. The charge-conjugation partner of fig. 1 ($A_{HG \dots ED}$).

i, j, \dots, k, l, m to be of different flavors. This specifies the flavor indices of the external mesons, and their location in the N^2 multiplets is therefore determined. This defines a physical process, which has the property that the only possible order that can contribute to it is that of fig. 1. If we view fig. 1 as representing the full $b = 1$ amplitude with the specified order, we can define it as the $b = 1$ approximation to the physical process we have just specified. Therefore, the behavior of the quark-diagram amplitude of fig. 1 as a function of the external momenta and helicities is actually determined by the behavior of the amplitude of a physical process (in the $b = 1$ approximation).

Let us denote the amplitude of fig. 1 by $A_{DE \dots GH}$ (only the cyclic order is relevant). We apply charge-conjugation transformation to the physical process that defines it. The state $|D_{i,m}\rangle$ for example, becomes $C_D |D_{m,i}\rangle$. Up to the $C_D \dots C_H$ factors, we get a new process, which can define the amplitude $A_{HG \dots ED}$ of fig. 2. This new amplitude corresponds to the same external multiplets and the same momenta and helicities, as in $A_{DE \dots GH}$. The only difference is that the order is reversed. Namely, the two amplitudes correspond to two different terms among the $(n - 1)!$ terms. By charge-conjugation invariance we get

$$A_{HG \dots ED} = [C_D C_E \dots C_G C_H] A_{DE \dots GH} \tag{2}$$

where $p_D, h_D, \dots, p_H, h_H$ are the same in both amplitudes. We have seen that the

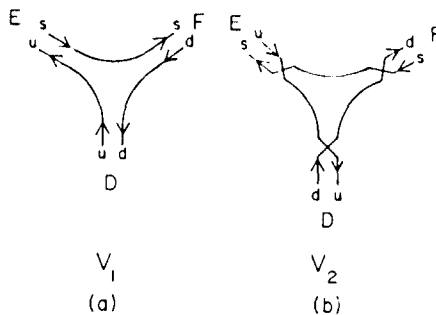


Fig. 3. The two components of the vertex.

$b = 1$ amplitude has $(n - 1)!$ terms. Each can be defined by a different choice of a physical process. They are related in pairs by eq. (2). When a process is forbidden by charge-conjugation, these amplitudes cancel in pairs. (Namely, if one amplitude contributes to this process the one with the reverse order must also contribute.) The generalization of eq. (2) for an amplitude with $b > 1$ is discussed in appendix C.

Let us now consider the three-point vertex of the multiplets D, E, and F. The $b = 1$ amplitude has two terms, as in fig. 3. We denote the A_{DEF} term by V_1 and the A_{FED} by V_2 . Let us consider the decay process $D \rightarrow EF$. The V_1 amplitude can be defined as the $b = 1$ amplitude of the physical process $D_{u\bar{d}} \rightarrow E_{u\bar{s}} F_{s\bar{d}}$. We use the center-of-mass frame of D, and take the initial state to be $|J, M\rangle$ where J is the spin of D. By angular-momentum conservation

$$V_1^{h_E, h_F}(\theta, \phi) \propto [\mathcal{D}_{M\mu}^J(\phi, \theta, -\phi)]^*,$$

where $\mu = \mu_E - \mu_F$ and $\Omega = \{\theta, \phi\}$ is the direction of E. If we interchange the directions of E and F (namely $\Omega \rightarrow -\Omega = \{\pi - \theta, \pi + \phi\}$), we get for $M = 0$,

$$V_1^{h_E, h_F}(\Omega) = \tau V_1^{h_F, h_E}(-\Omega), \quad (3)$$

where $\tau = (-)^J$ is the signature of D.

The point we want to emphasize is that since the quark diagram V_1 is defined in terms of the amplitude of a physical process, it must satisfy all the constraints imposed on this process by the various symmetries. As we have just seen the angular dependence of the decay, and therefore the symmetry under $\Omega \rightarrow -\Omega$, is governed by the spins of the particles. The relation between $V_1^{h_E, h_F}(\Omega)$ and $V_1^{h_F, h_E}(-\Omega)$ is determined by the parity of the particles. The relation between the various physical regions ($D_{u\bar{d}} \rightarrow E_{u\bar{s}} F_{s\bar{d}}$, $E_{s\bar{u}} \rightarrow F_{s\bar{d}} D_{d\bar{u}}$ and $F_{d\bar{s}} \rightarrow D_{d\bar{u}} E_{u\bar{s}}$) is determined by the crossing properties of the physical amplitude. To derive all these properties of the V_1 amplitude, we do not have to draw any quark diagram. These are just the properties of the amplitude for the physical process we have chosen.

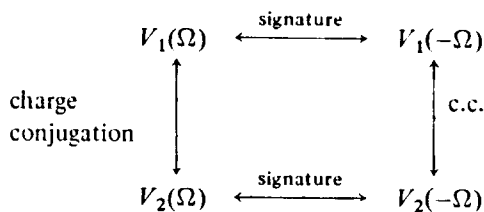
The quark diagram V_2 (fig. 3b) is defined as the $b = 1$ amplitude for a different physical process: $D_{d\bar{u}} \rightarrow E_{s\bar{u}} F_{d\bar{s}}$. The multiplets D, E, and F are the same as in the physical process that defines the V_1 amplitude, but we have chosen different members of the multiplets. The angular-momentum, parity and crossing properties of V_2 are again determined by the properties of the physical amplitude that defines it. The relation between V_1 and V_2 is determined by charge-conjugation invariance, since the two defining physical processes are related by charge conjugation. Applying eq. (2) to the three-point function we get

$$V_2^{h_E, h_F}(\Omega) = C_D C_E C_F V_1^{h_E, h_F}(\Omega), \quad (4)$$

where in both sides Ω is the direction of particle E.

We summarize the different roles of charge conjugation and signature in the fol-

lowing table:



Charge conjugation relates two different quark diagrams. In one of them the quark of D goes to E, and in the other it is the antiquark of D which goes to E. We get one diagram from the other by twisting the quark antiquark lines of each external meson. Signature, on the other hand, has nothing to do with quark diagrams. It relates the values of the amplitude of a given physical process at different points in momentum space. (One source of confusion is the fact that V_2 is sometimes drawn as in fig. 4. The fact that in fig. 3a particle E is drawn to the left, whereas in fig. 4 it is drawn to the right, has nothing to do with right and left in real (or momentum) space. V_1 in fig. 3a and V_2 in fig. 4 are still related by charge conjugation, as in eq. (4), provided that particle E is in the same direction, Ω , in momentum space.) If E and F are identical multiplets, charge conjugation is related to signature. This will be discussed in appendix B.

The minimal number of flavors, N , which is required in order to define V_1 and V_2 in terms of physical reactions, is two. (For example, $D_{u\bar{u}} \rightarrow E_{u\bar{d}} F_{d\bar{u}}$ defines the V_1 amplitude.) For $N = 1$, the decomposition of the vertex function of $D_{u\bar{u}} \rightarrow E_{u\bar{d}} F_{u\bar{u}}$ into the two components V_1 and V_2 is unambiguous if the underlying dynamics is such that one can define the amplitude for the quark of D to go to E. For example, if we have field theory with quarks, the sum of all Feynman diagrams for the 3-meson vertex, in which the quark line of D goes to E, will define the V_1 part of the vertex. In that case, eq. (4) is still valid.

We now proceed to the 4-point function. We consider the Regge-exchange diagram of fig. 5a. Since this particle diagram has two vertices, and each one can be either V_1 or V_2 , there are four quark diagrams: $A_1 = A_{EFGH}(\frac{V_1}{V_1})$, $\bar{A}_1 = A_{HGFE}(\frac{V_2}{V_2})$,

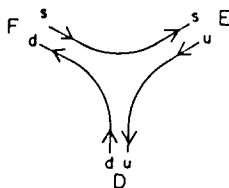


Fig. 4. V_2 in a different notation.

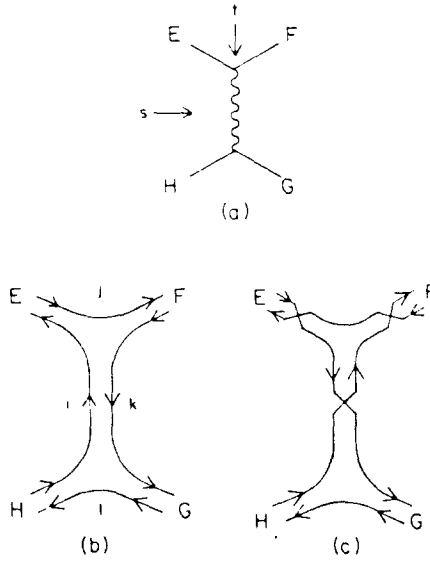


Fig. 5. Untwisted and twisted diagrams (in the quark line sense).

$A_2 = A_{EFGH}(\frac{F}{1}, \frac{2}{1})$ and $\bar{A}_2 = A_{FHGE}(\frac{F}{1}, \frac{1}{2})$. Using eq. (2), they are related in pairs:

$$\bar{A}_1 = C_E C_F C_G C_H A_1, \quad \bar{A}_2 = C_E C_F C_G C_H A_2, \quad (5)$$

and therefore we consider only A_1 and A_2 . As before, if N is large enough, we can define the quark diagram A_1 in terms of the physical process $E_{ji} H_{il} \rightarrow F_{jk} G_{kl}$ where i, j, k, l are four different flavors. (If we start from the particle diagram (5a) then two flavors are sufficient, since the diagram is constructed from vertices and propagators.) We now consider the contribution to A_1 of a single-Regge exchange, R , with well-defined signature, τ , and charge conjugation C . In a symbolic notation, this contribution is given by

$$A_1^R = \beta [(-s)^\alpha + \tau(-u)^\alpha]. \quad (6)$$

The first term, $\beta(-s)^\alpha$, is represented in ref. [5] by fig. 6a. It is the part of the amplitude that has only s -channel discontinuity. The twisted diagram (fig. 6b) represents the amplitude we get by replacing $s \rightarrow u$ in the untwisted diagram, namely $\beta(-u)^\alpha$. We refer to this twist as the signature twist. *A priori*, it has nothing to do with quark diagrams. The arguments of ref. [5] do not involve the concept of quarks or of internal symmetries. Only general properties of physical amplitudes are used. Since the quark diagram A_1 can be defined in terms of a physical process, we could apply these arguments here. Generally speaking, the signature twist of ref. [5] rep-

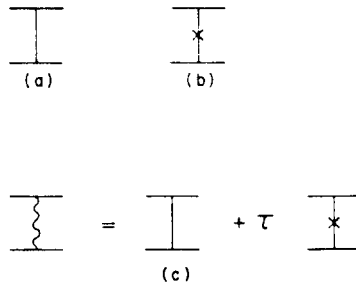


Fig. 6. Untwisted and twisted diagrams (in the signature sense).

resents a transformation in momentum space in which all the invariants across the twisted reggeon (such as s for the 4-point function) get asymptotically a minus sign ($s \leftrightarrow -s$). This signature structure has nothing to do with planarity, since the reggeons are not necessarily exchange-degenerate. The relative sign between a signature-twisted and a signature-untwisted diagram is determined, as in eq. (6) by the signature of the exchanged reggeon. Diagrammatically, eq. (6) is represented in ref. [5] by fig. 6c.

The quark diagram A_2 is related to A_1 by replacing the upper V_1 vertex by V_2 . Using eq. (4) we get

$$A_2^R = CC_F D_F A_1^R. \tag{7}$$

The contribution of a single-Regge exchange to A_1 and to A_2 has always both s -channel and u -channel discontinuities. When we add another Regge exchange, \tilde{R} with signature $\tilde{\tau}$ and charge conjugation \tilde{C} we get

	A_1	A_2
$R(\tau, C)$	$\beta[(-s)^\alpha + \tau(-u)^\alpha]$	$CC_F C_F \beta[(-s)^\alpha + \tau(-u)^\alpha]$
$\tilde{R}(\tilde{\tau}, \tilde{C})$	$\tilde{\beta}[(-s)^{\tilde{\alpha}} + \tilde{\tau}(-u)^{\tilde{\alpha}}]$	$\tilde{C}C_F C_F \tilde{\beta}[(-s)^{\tilde{\alpha}} + \tilde{\tau}(-u)^{\tilde{\alpha}}]$

When we take the contribution of both reggeons into account, we get $A_1 = A_1^R + A_1^{\tilde{R}}$ and $A_2 = A_2^R + A_2^{\tilde{R}}$. In order to eliminate the u -channel discontinuity from A_1 , we must have $\tilde{\tau} = -\tau$ together with exchange degeneracy: $\tilde{\alpha} = \alpha, \tilde{\beta} = \beta$. In order to eliminate at the same time the s -channel discontinuity from A_2 , we must have $\tilde{C} = -C$. Therefore, in the planar level ($b = 1, h = 0$), where we expect A_1 and A_2 to have only the s - (u -) channel discontinuities, each trajectory has to be exchange-degenerate with another trajectory which has the opposite τ and the opposite C .

However, no relation between C and τ is required. The leading trajectory happens to have $C\tau = +1$, but the same pattern of exchange degeneracy can be achieved with a pair of $C\tau = -1$ trajectories. At the planar level we get

$$(l = 0 \text{ only}) \quad A_1 = \overline{\text{---}} \quad , \quad A_2 = C_E C_F (C\tau) \overline{\text{---}} \quad . \quad (8)$$

A_1 and A_2 are the untwisted and twisted quark diagrams. The twist in A_2 is a quark-line twist, which is related to charge conjugation. Only at the planar level they correspond to the signature-untwisted and signature-twisted diagrams (up to the $C_E C_F C\tau$ factor).

The generalization to a multiperipheral process with n produced mesons, is straight forward. For a given set of exchanges, the amplitude has 2^{n-1} terms, as in ref. [5] (each exchanged reggeon can be signature-twisted or not). There are 2^{n-1} pairs of quark diagrams. Each quark diagram can be defined in terms of the $b = 1$ amplitude of a physical process (if $N \geq 2$). Only if we assume exchange degeneracy, the two types of twisted diagrams coincide, as is shown in ref. [2].

For multiperipheral diagrams with a baryon exchange, the situation is very similar. The different quark diagrams, as defined in ref. [6], can be defined in terms of physical amplitudes for $N \geq 3$. The angular-momentum (signature), parity and

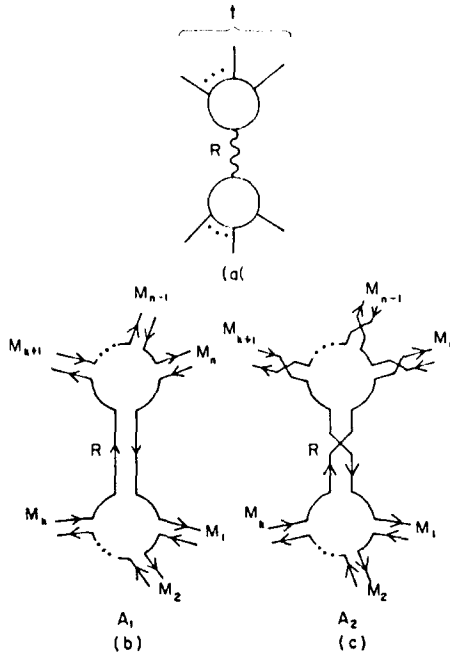


Fig. 7. The quark-line twist for one reggeon exchange.

crossing properties of the various quark diagrams are governed by those of the corresponding physical processes. The relation between the different quark diagrams is governed by the permutation symmetry of the exchanged baryon under the interchange of the flavor indices. For mesons, the permutation symmetry is related to charge conjugation (appendix A). For baryons, the permutation symmetry is related to their $SU(N)$ multiplets [6]. A transformation from particle diagrams to quark diagrams was constructed in ref. [6]. The particle diagrams carried $SU(N)$ indices. A similar transformation for mesons is constructed in appendix C. The particle diagrams carry charge-conjugation indices.

We now consider the n -point function for mesons, in the single-Regge limit (fig. 7). The reggeon R has charge conjugation C_R . A_1 and A_2 are two quark diagrams that contribute in that limit. If N is large enough they can be defined, as usual, in terms of physical processes. The order of the external mesons on the single boundary of A_1 is $M_1 M_2 \dots M_k M_{k+1} \dots M_n$. For A_2 the order is $M_1 M_2 \dots M_k M_n \dots M_{k+1}$. We want to relate the two amplitudes for the same values of momenta and helicities (namely, p_i, h_i are the same for A_1 and A_2 , for $i = 1, \dots, n$). We use factorization and apply eq. (2) to the upper vertex. The result is

$$(fig. 7) \quad A_2 = C_R C_{k+1} \dots C_n A_1. \tag{9}$$

Actually, the quark diagrams A_1 and A_2 are given by summing over R . To project the part of the amplitude which has a definite charge conjugation, C , in the t -channel we should take the following combination:

$$A^C = A_1 + C C_{k+1} \dots C_n A_2. \tag{10}$$

I would like to thank F. Capra, G. Chew and P. Hoyer for useful discussions.

Appendix A

The charge-conjugation convention

In a model, in which mesons are bound states of a quark and an antiquark, the $i\bar{j}$ state of the multiplet D with momentum p and helicity h , can be written as:

$$|D_{i\bar{j}}(p, h)\rangle = \sum_{\substack{r,s \\ \alpha,\beta}} \frac{1}{\sqrt{N_c}} \delta_{rs} f_{\alpha\beta}^{D(p,h)} b_{ir\alpha}^\dagger d_{js\beta}^\dagger |0\rangle. \tag{A.1}$$

b^\dagger and d^\dagger are creation operators of quarks and antiquarks. They satisfy $b^\dagger d^\dagger = -d^\dagger b^\dagger$. The indices i and j are flavor indices, r and s are color indices and N_c is the number of colors. The meson is assumed to be in a color-singlet state; α and β are indices which specify both the momentum and the helicity of the quark; $f_{\alpha\beta}^{D(p,h)}$ is the

wave function of meson $D(p, h)$ in momentum-spin space. It has a definite permutation symmetry: $f_{\alpha\beta}^D = P_D f_{\beta\alpha}^D$ where P_D is either $+1$ or -1 . In the $L - S$ scheme, $P_D = (-)^L (-)^{S+1}$ where L is the orbital momentum and S is the total spin of the quark-antiquark pair.

In a quark-line notation, the state $b_{i r \alpha}^+ d_{j s \beta}^+ |0\rangle$ can be represented as $\begin{array}{c} i r \alpha \text{ ---} \\ | \\ j s \beta \text{ ---} \end{array}$. Note that this state is minus the $\begin{array}{c} i s \beta \text{ ---} \\ | \\ i r \alpha \text{ ---} \end{array}$ state, since b^+ and d^+ anticommute. This is the reason why we prefer to draw the quark and antiquark lines of the external mesons in a definite order (as in fig. 3).

We now apply the charge-conjugation operator C to the state in eq. (A.1). We use $C b_{i r \alpha}^+ C^{-1} = d_{i r \alpha}^+$ to get in a standard way

$$C |D_{ij}^-(p, h)\rangle = -P_D |D_{ji}^-(p, h)\rangle, \quad (\text{A.2})$$

which is eq. (1) with $C_D = -P_D$.

Appendix B

Identical particles

If E and F in fig. 3 stand for the same multiplet (we do not require that the flavor indices of E and F are the same, so the two mesons are not necessarily identical), we can get an extra relation between V_1 and V_2 . $V_1^{j_1, h_1, j_2, h_2}(p_1, p_2)$ was defined as the $b = 1$ amplitude of $D_{u\bar{d}} \rightarrow E_{u\bar{s}}(p_1, h_1) F_{s\bar{d}}(p_2, h_2)$. By $SU(N)$ invariance we can change the flavor labels of fig. 3b and define $V_2^{j_1, h_1, j_2, h_2}(p_1, p_2)$ as the $b = 1$ amplitude of $D_{u\bar{d}} \rightarrow E_{s\bar{d}}(p_1, h_1) F_{u\bar{s}}(p_2, h_2)$. Since $E_{u\bar{s}} = F_{u\bar{s}}$, we get

$$(E = F) \quad V_1^{j_1, h_1, j_2, h_2}(p_1, p_2) = V_2^{j_2, h_2, j_1, h_1}(p_2, p_1). \quad (\text{B.1})$$

Using eq. (4) we get

$$(E = F) \quad V_1^{j_1, h_1, j_2, h_2}(p_1, p_2) = C_D V_1^{j_2, h_2, j_1, h_1}(p_2, p_1). \quad (\text{B.2})$$

Choosing $h_1 = h_2$ and comparing with eq. (3), we see that if $V_1^{j_1, h, j_2, h}(p_1, p_2) \neq 0$, the meson D must satisfy $C = \tau$. Note that if $C_D = -1$ the V_1 amplitude is antisymmetric under interchanging $p_1 h_1$ and $p_2 h_2$. This does not violate Bose symmetry. When the two mesons are really identical (all flavor indices are the same) both V_1 and V_2 contribute, and their sum vanishes by eq. (4).

Appendix C

Quark diagram \leftrightarrow particle-diagram transformation

We now construct quark diagrams from particle diagrams as in ref. [2]. We consider the most general particle diagram for mesons, with 3-point vertices only. To

each vertex we assign an arbitrary cyclic order for the three mesons and define V_1 as the vertex component in which the arrow follows the order. The building blocks of the diagram are the vertices and the propagators. We assume that both of them are on the $b = 1$ level. This guarantees the OZI rule for the vertex, and the N^2 degeneracy for the propagator (which is needed if we want the flavor indices to flow continuously along the quark lines [2]). Since the vertex is of one boundary, eq. (4) is valid. We denote the charge conjugation of the j th external meson by C_j ($j = 1, \dots, n$). We first consider a given set of internal mesons, with charge conjugation C_i ($i = 1, \dots, I$). The particle amplitude, B , which corresponds to this particular set of exchanges will be defined as follows: Draw the quark diagram which has only V_1 vertices. Give any quark line one flavor index (do not sum over the N flavors in a quark loop). Calculate the amplitude as a product of vertices and propagators. (We assume that the rules for calculating particle diagrams in terms of vertices and propagators are given.)

There are 2^v different quark diagrams, where v is the number of vertices (each vertex can be V_1 or V_2). Let σ_k be the label for the k vertex. $\sigma_k = +$ corresponds to V_1 and $\sigma_k = -$ to V_2 . $\{\sigma\} = \{\sigma_1, \dots, \sigma_v\}$ is the label of the quark diagram. The B amplitude has only V_1 vertices. If we replace the V_1 of the k vertex by V_2 , the amplitude is multiplied by $\kappa_k = C_{D_k} C_{E_k} C_{F_k}$ where D_k, E_k and F_k are the three mesons of the vertex k . Therefore, the contribution of the given set of internal mesons to the quark diagram $A_{\{\sigma\}}$ is given by B times κ_k for each vertex k with V_2 ($\sigma_k = -$). We observe that two different sets of exchanges which have the same $\{\kappa\} = \{\kappa_1 \dots \kappa_v\}$, give exactly the same relations between the various quark diagrams. We sum the B amplitudes of all the set of exchanges with the same $\{\kappa\}$, to get $B^{\{\kappa\}}$. (The sum includes different sets which have the same C_i for each propagator, but differ by other quantum-numbers. It also includes different sets with different C_i . For each particle loop we can change the sign of all the C_i around the loop, without changing the $\{\kappa\}$. Note that for any intermediate state in the diagram, the product $\prod_r C_r$, where r runs over the particles of the intermediate state, is determined by $\{\kappa\}$.) The contribution of $B^{\{\kappa\}}$ to the quark diagram $A_{\{\sigma\}}$ is given by

$$A_{\{\sigma\}}^{\{\kappa\}} = M_{\sigma_1 \kappa_1} \dots M_{\sigma_v \kappa_v} B^{\{\kappa\}}, \tag{C.1}$$

where

$$M = \begin{matrix} & + & - \\ + & \begin{matrix} | & | \\ \hline 1 & 1 \\ \hline \end{matrix} & \\ - & \begin{matrix} | & | \\ \hline 1 & -1 \\ \hline \end{matrix} & \end{matrix} . \tag{C.2}$$

The quark-diagram $A_{\{\sigma\}}$ is defined to be

$$A_{\{\sigma\}} \equiv \sum_{\{\kappa\}} A_{\{\sigma\}}^{\{\kappa\}} \tag{C.3}$$

and the transformation from the particle amplitudes B to the quark diagrams A is

$$A_{\{\sigma\}} = \sum_{\{\kappa\}} M_{\sigma_1 \kappa_1} \dots M_{\sigma_v \kappa_v} B^{\{\kappa\}}. \quad (\text{C.4})$$

The inverse transformation is

$$B^{\{\kappa\}} = \frac{1}{2^v} \sum_{\{\sigma\}} M_{\sigma_1 \kappa_1} \dots M_{\sigma_v \kappa_v} A_{\{\sigma\}}. \quad (\text{C.5})$$

The $A_{\{\sigma\}}$ are defined without summing over flavor indices. When this sum is performed, each quark diagram is multiplied by a factor of N for each quark loop. To calculate a physical process (given in the quark basis) one has to add all the $A_{\{\sigma\}}$ that are consistent with the quark indices without any additional coefficient.

The number of particle-amplitudes is 2^{v-l} , since the $\kappa_1, \dots, \kappa_v$ are constrained by the relation $\kappa_1 \cdot \dots \cdot \kappa_v = C_1 \cdot \dots \cdot C_n$. There are 2^v quark-diagrams. They are related in pairs according to

$$A_{\{-\sigma\}} = C_1 \dots C_n A_{\{\sigma\}}. \quad (\text{C.6})$$

This is the generalization of eq. (2) to any number of boundaries. $A_{\{-\sigma\}}$ and $A_{\{\sigma\}}$ are related by a $V_1 \leftrightarrow V_2$ transformation, which interchanges the roles of quarks and antiquarks. Therefore, they have the same boundary structure, but the order of external mesons along each boundary is the opposite.

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